Review: Math 3300: Foundations of Abstract Algebra

Goal: To review the material traditionally covered in Foundations of Abstract Algebra course (Math 3300).

Structure of the Module: The module is divided in 10 parts (five in **Part 1** and five in **Part 2** of this module). Each of the parts has a soft deadline attached to it with aim to guide students in achieving the objectives and reviewing the material in a timely manner.

Each part (except the last one Practice - GRE Problems) starts with a list of objectives that need to be achieved after reviewing a particular topic (listed in **Topics Covered**) and has two assignments attached to it.

Topics Covered:

Part 1 (Week 1)

Day 1: Preliminaries (review of Integers and Equivalent Relations)

Day 2: Introductions of Groups

Day3: Groups

Day 4: Finite Groups and Subgroups

Day 5: Cyclic Groups and Permutations Groups

Part 2 (Week 2)

Day 1: Cosets and Lagrange's Theorem

Day 2: Group Homomorphisms and Isomorphisms

Day 3: Normal Subgroups and Factor Groups

Day 4: Introduction to Ring Theory

Day 5: Practice - GRE Problems

Recommended Textbook: The topics listed above follow closely the material covered in Chapters 0 -10 (excluding Chapter 8) in Gallian's book:

J. Gallian, *Abstract Algebra*, *Contemporary Abstract Algebra*, 8th Edition, by Joseph A. Gallian

(Note that any edition of this book will cover the same material)

Additional Reference Textbooks (can be found in the Library): The topics listed above are topics covered in any traditional Abstract Algebra book. Attached is a list of books that can be found in the UVU Library.

Integers and Equivalent Relations

Chapter 0: Preliminaries (Integers and Equivalence Relations)

Upon successful review of Chapter 0 (Gallian's textbook), student(s) should be able to

- define and use/apply the following concepts: a divisor and a multiple of an integer, a prime integer, greatest common divisor (*gcd*), relatively prime integers, least common multiple (*lcm*)

- state and apply the Well Ordering Principle, the Division Algorithm, properties of *gcd*, Euclid's Lemma, and the Fundamental Theorem of Arithmetic

- work with modular arithmetic

- work with complex numbers and use/apply their properties

- define and use/apply the following concepts: equivalence relation, partition, equivalence classes

- define and use/apply the following concepts: a function (mapping), composition of functions, a one-to-one function, an onto function, and be familiar with their properties.

Introduction to Groups

Chapter 1: Introduction to Groups

Upon successful review of Chapter 1 (Gallian's textbook), student(s) should be able to

- define, identify, and work with the Dihedral groups

- construct a Cayley table of finite groups

Groups

Chapter 2: Groups

Upon successful review of Chapter 2 (Gallian's textbook), student(s) should be able to

- state definitions of a binary system and a group

- prove that a set is a binary system/group under a given operation

- recognize, prove, and work with variety of groups (Z, R, Q, C, Z_n, GL(2,R), SL(2,R), U(n), etc.)

- prove various elementary properties of groups (uniqueness of identity and inverses, socks-shoes property, solving equations, etc.)

- use elementary group properties to prove more advanced group results

Finite Groups and Subgroups

Chapter 3: Finite Subgroups; Subgroups

Upon successful review of Chapter 3 (Gallian's textbook), student(s) should be able to

- state the definition of an order of a group/element, subgroup, center of a group, centralizer of an element in a group

- apply these definitions on a concrete examples

- prove group results using these concepts

- state and use Subgroup Tests in various settings

- prove, find, and work with various examples of subgroups (cyclic subgroup, center of a group, centralizer of an element in a group

Cyclic Groups and Permutations Groups

Chapter 4: Cyclic Groups

Upon successful review of Chapter 4 (Gallian's textbook), student(s) should be able to

- work with cyclic groups (know examples of cyclic groups, prove that a group is cyclic, find which of the groups considered in Chapter 2 are cyclic, etc.)

- find generators of a cyclic group, and in particulars generators of Z_n

- familiar with various results about orders of elements in a cyclic group

- state and use the Fundamental Theorem of Cyclic Groups

- (optional) use the \phi function and Subgroup lattice of Z_n groups

Chapter 5: Permutation Groups

Upon successful review of Chapter 4 (Gallian's textbook), student(s) should be able to

- state and use/apply definitions of a permutation of a set A, permutation group of A, symmetric groups of degree n, and alternating group of degree n

- use cyclic notation of permutations

- work with various properties of permutations (product of disjoint cycles, disjoint cycles commute, order of a permutation, even/odd permutation

Cosets and Lagrange's Theorem

Chapter 7: Cosets and Lagrange's Theorem

Upon successful review of Chapter 7 (Gallian's textbook), student(s) should be able to

- state, apply, and use definition of a coset of a subgroup in a group
- find cosets of various subgroups
- prove basic coset properties
- use basic properties of cosets to prove more advanced group results
- state, prove, and use/apply Lagrange's theorem and its Corollaries
- state and use the Classification of groups of order 2p

Group Homomorphisms and Isomorphisms

Chapter 6: Group Isomorphisms

Upon successful review of Chapter 6 (Gallian's textbook), student(s) should be able to

- state and apply the definition of group isomorphisms, automorphisms, inner automorphisms induced by an element a
- show that a map is an isomorphism, automorphism, inner automorphism
- prove that two groups are isomorphic
- state, prove, and apply the Cayley's Theorem
- state, prove, and use/apply various isomorphism properties
- define and find the Automorphism group and Inner Automorphism group

Chapter 10: Group Homomorphism

Upon successful review of Chapter 10 (Gallian's textbook), student(s) should be able to

- state and apply the definition of group homomorphisms, kernel of a homomorphism
- show that a map is an homomorphism and find its kernel
- state, prove, and use/apply various homomorphism properties
- state, prove, and use/apply the First Isomorphism Theorem and its Corollaries

Normal Subgroups and Factor Groups

Chapter 9: Normal Subgroups and Factor Groups

Upon successful review of Chapter 9 (Gallian's textbook), student(s) should be able to

- state and apply the definition of a normal subgroup, a factor group

- prove that a subgroup is normal using the definition as well as the Normal Subgroups Test

- form factor groups in various group setting

- prove various applications of Factor Groups (G/Z Theorem, Cauchy's Theorem for Abelian Groups)

Introduction to Ring Theory

Chapter 12, 13: Introductions to Rings, Integral Domains, and Fields

Upon successful review of Chapter12 (Gallian's textbook), student(s) should be able to

- state and use/apply the definition of a ring, unit, unity, zero divisor, integral domain, and field

- provide various examples of rings

- state and prove various elementary properties of rings